MATH-5C TEST 1 v1 (Chapter 12, 13.1, 13.2, ~~13.4i~~)
Spring 2024

100 points **NAME:**________________________

- **Full instructions on Canvas**
- **I expect clear and legible presentations with words of explanation . No credit given if work is not shown. Only methods used in this class are accepted**
- **Exact answers expected unless otherwise specified.**
- **No resources may be used other than your page of notes.**

(1) Find the equatin of the plane containing the point (0,-4,5) and orthogonal to the line

$$\begin{cases} x = 3+t \\ y = 7+2t \\ z = -4+3t \end{cases}$$

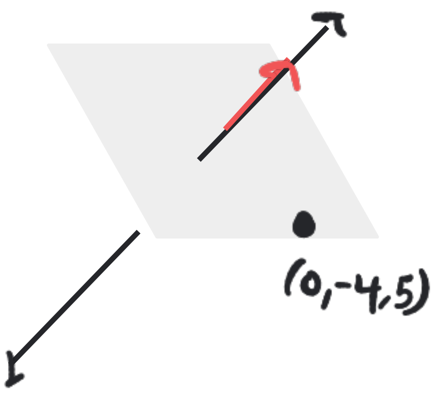
(4 points)

Plane

Point $(0,-4,5)$

$\vec{v} = \langle 1,2,3 \rangle$

$\vec{n} = \vec{v} = \langle 1,2,3 \rangle$



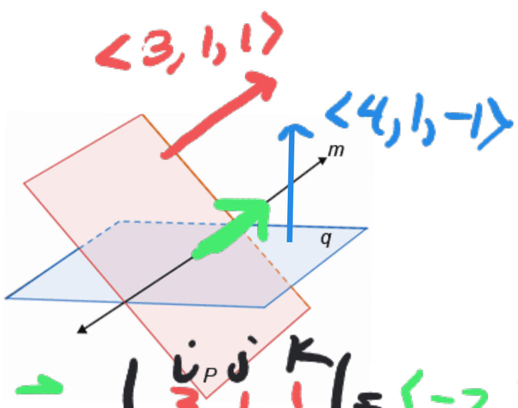
$1(x-0)+2(y+4)+3(z-5)=0$

$$\boxed{x+2y+3z=7}$$

(2) Find the equations of the line of intersection of the planes: $3x+y+z=2$ and $4x+y-z=6$

(9 points)

Geometrically



$\vec{v} = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 4 & 1 & -1 \end{vmatrix} = \langle -2,7,-1 \rangle$

Point: any point satisfying both planes: $(4,-10,0)$

Algebraically

$\begin{cases} 3x+y = 2-z \\ 4x+y = 6+z \end{cases}$

$x = 4+2z$

$y = 6+z-4(4+2z)$

$y = -10-7z$

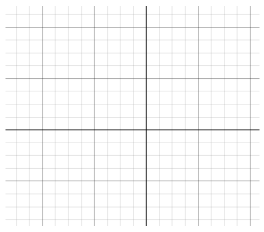
$$\boxed{\begin{cases} x = 4+2t \\ y = -10-7t \\ z = t \end{cases}}$$

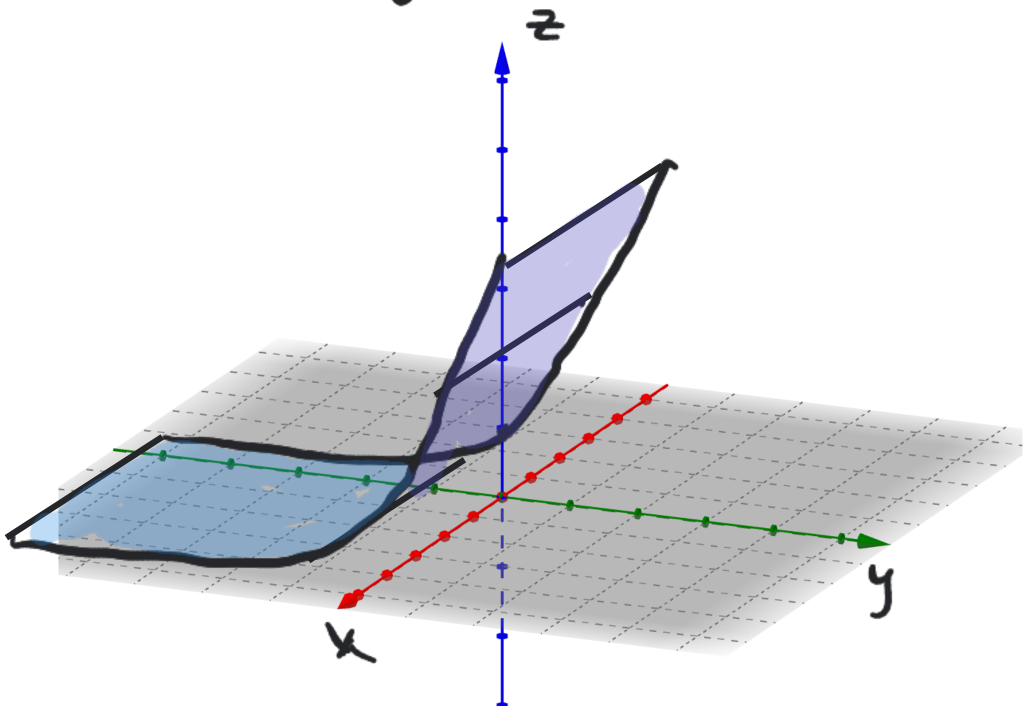
(3) On separate axes, sketch a graph of the following **surfaces** in R^3. Name the surface and give pertinent information such as traces. Label axes. Show scale clearly on axes and one important cross section. Graphing software or calculators not allowed.(Use small grids for traces if needed)

(14 points)

(a) $z = e^y$

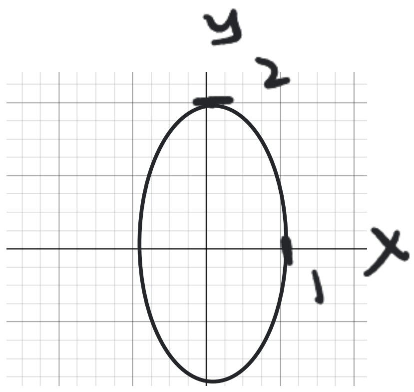
Name of surface: cylinder

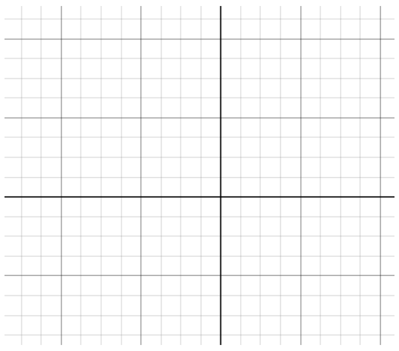




(b) $z = 4x^2 + y^2$

Name of surface: paraboloid

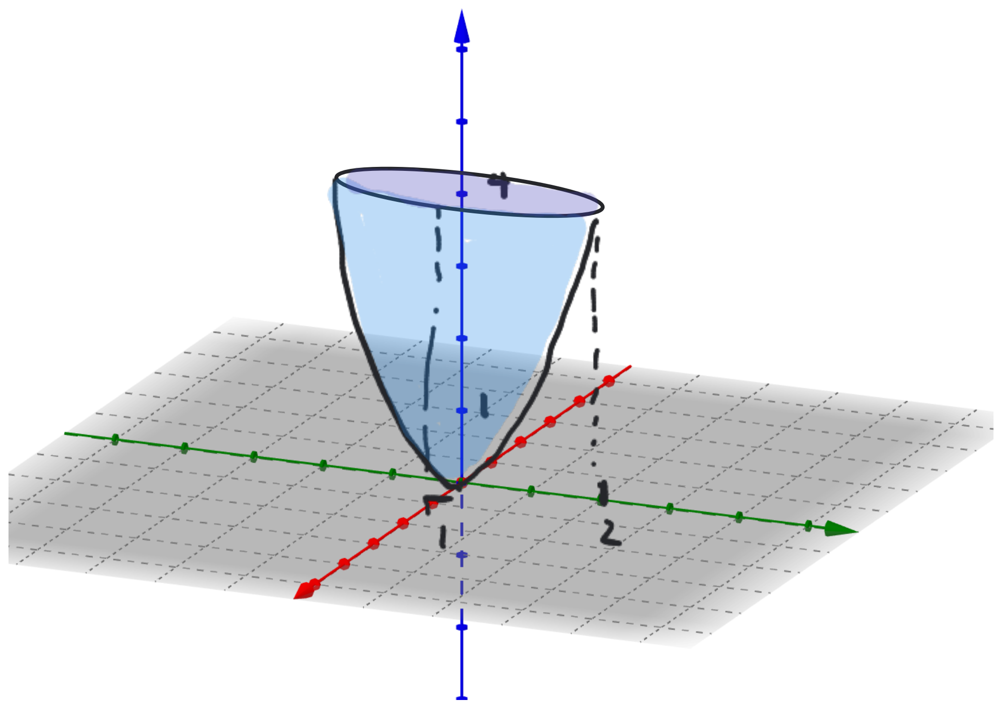




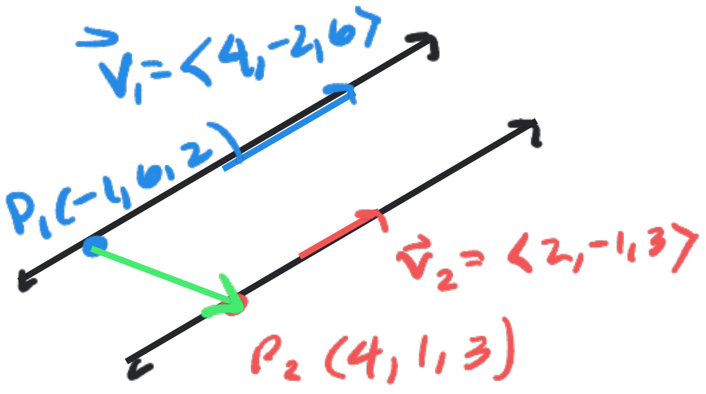
$z = 4$

$4 = 4x^2 + y^2$

$1 = x^2 + \frac{y^2}{4}$



(4) Find an equation of the plane containing the lines $\overset{L_1}{\begin{cases} x = -1+4t \\ y = 6-2t \\ z = 2+6t \end{cases}}$ and $\overset{L_2}{\begin{cases} x = 4+2s \\ y = 1-s \\ z = 3+3s \end{cases}}$ (9 points)



Lines are parallel since $\vec{v}_1 = 2\vec{v}_2$
(If you cross them, you'll get zero)

Form vector $\overrightarrow{P_1P_2} = \langle 5,-5,1\rangle$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -5 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \langle -14,-13,5\rangle$$

* can check

point $(4,1,3)$

Plane: $-14(x-4)-13(y-1)+5(z-3)=0$

$$\boxed{-14x-13y+5z+54=0}$$

* can check that both lines lie in plane

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(5) Find the equations for the line tangent to the curve $\vec{r}(t) = \left\langle \sin(2\pi t), t^3, \sqrt{t+7}\right\rangle$ at the t=2. *(9 points)*

$\vec{r}\,'(t) = \left\langle 2\pi\cos 2\pi t,\ 3t^2,\ \dfrac{1}{2\sqrt{t+7}}\right\rangle$

$\vec{v} = \vec{r}\,'(2) = \langle 2\pi, 12, 1/6\rangle$

point: $\vec{r}(2) = (0, 8, 3)$

Line: $\boxed{\begin{cases} x = 2\pi t \\ y = 8+12t \\ z = 3+\frac{1}{6}t \end{cases}}$

(6) Given the vectors $\mathbf{a} = \langle 4,1,-2 \rangle$ and $\mathbf{b} = \langle 3,0,1 \rangle$ find the following:

$\vec{b} = \langle 3,0,1 \rangle$

(4 points each)

a) $\mathbf{a} \times \mathbf{b}$ **(no partial credit here – easy to check)**

$\langle 1, -10, -3 \rangle$

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b) the angle between $\mathbf{a}$ and $\mathbf{b}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \frac{10}{\sqrt{24}\sqrt{10}} = \sqrt{\frac{10}{21}}$$

$\cos^{-1}\left(\sqrt{\frac{10}{21}}\right)$

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c) $\text{proj}_{\mathbf{a}}\mathbf{b}$

$$\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{10}{21}\vec{a}$$

$\left\langle \frac{40}{21}, \frac{10}{21}, \frac{-20}{21} \right\rangle$

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d) a unit vector in the direction of $\mathbf{b}$

$\left\langle \frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}} \right\rangle$

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e) a value for k such that < k, 3,-6 > is parallel to $\mathbf{a}$

$\vec{a} = 4\ 1\ -2$

$3\vec{a} = \langle K, 3, -6 \rangle$

$\Rightarrow K = 12$

12

(7) Intersections – SHOW WORK (12 points

(a) Does the curve $\vec{r}(t)=\langle t^2, \cos(\pi t), e^t\rangle$ contain the point (1, -1, 1)? ___no___

(b) Does the plane 3x-4y+2z=3 contain the point (3,2,1)?___yes___

(c) Does the plane x+5y-2z=4 contain the line $L\begin{cases} x=4t+1 \\ y=1 \\ z=1+2t \end{cases}$?___yes___

$4t+1+5(1)-2(1+2t)=4$

$4t+6-2-4t=4$

$4=4 \Rightarrow$ infinitely many solns.

(d) Does the curve $\vec{r}(t)=\langle 0, t, 2t-t^2\rangle$ intersect the surface $z=x^2+y^2$ at the point (0,1,1)___yes___?

$\vec{r}(1)=\langle 0,1,1\rangle$ so (0,1,1) on curve

$1=0^2+1^2$ so (0,1,1) on surface also

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(8). Find the distance between the point (3,0,6) and the plane $9x+2y+z=7$. *(7 points)*

$$d=\frac{|9(3)+2(0)+6-7|}{\sqrt{81+4+1}}=\frac{26}{\sqrt{86}}$$

(9) Match the following equations to the graphs below. (Two of the graphs have no match. Axes are standard orientation with x axis red, y axis green and z axis blue.) (8 points)

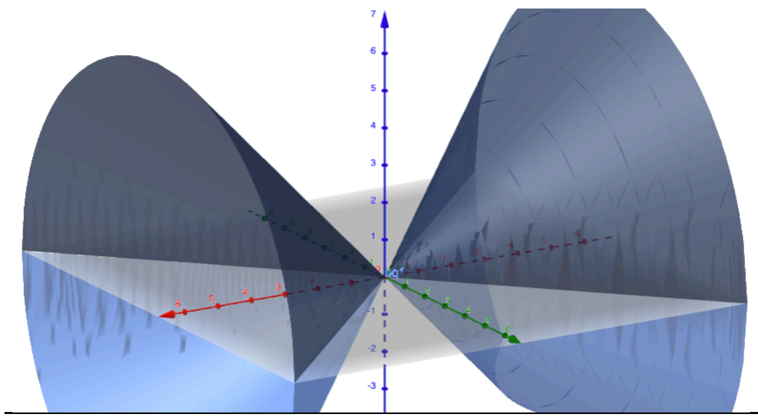
a) $x^2 - \frac{y^2}{4} + z^2 = 1$ ___II___

b) $z = \sin(x)$___V___

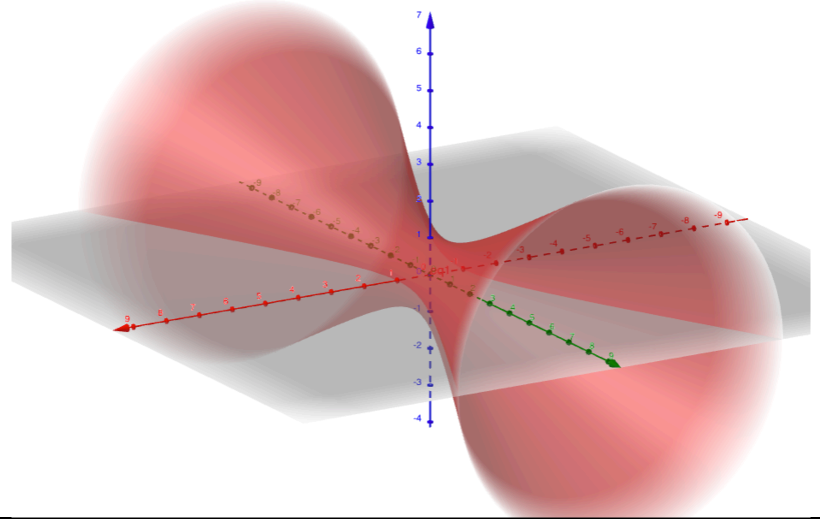
c) $z = \sin(x - y)$ ___IV___

d) $z = -\sqrt{x^2 + y^2}$ ___III___

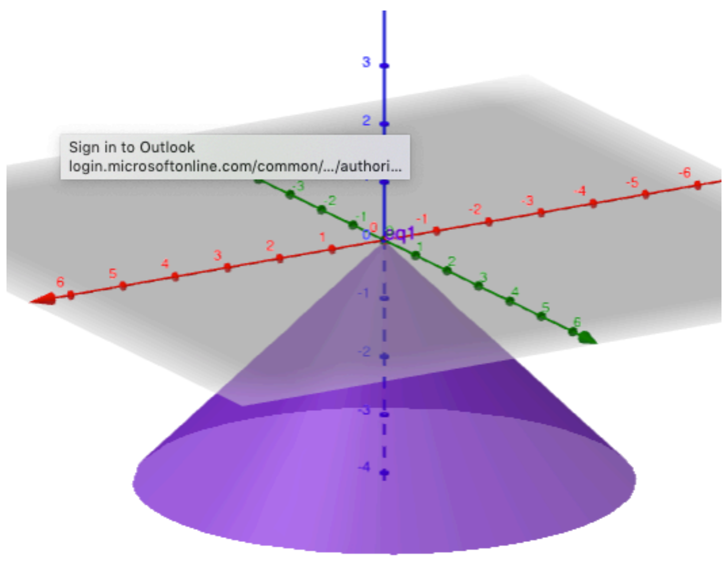
I



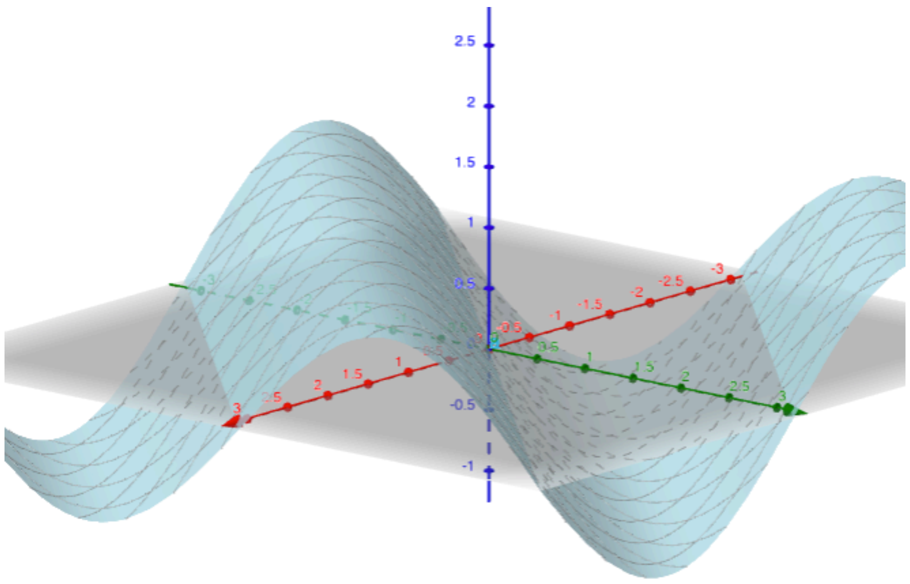
II



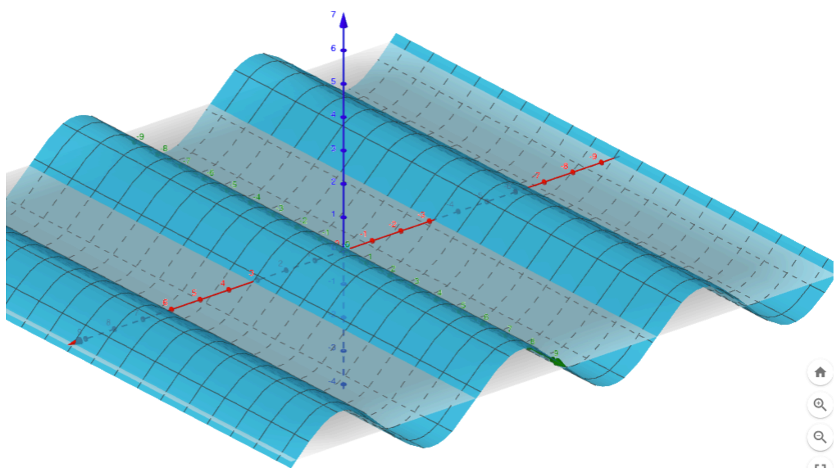
III



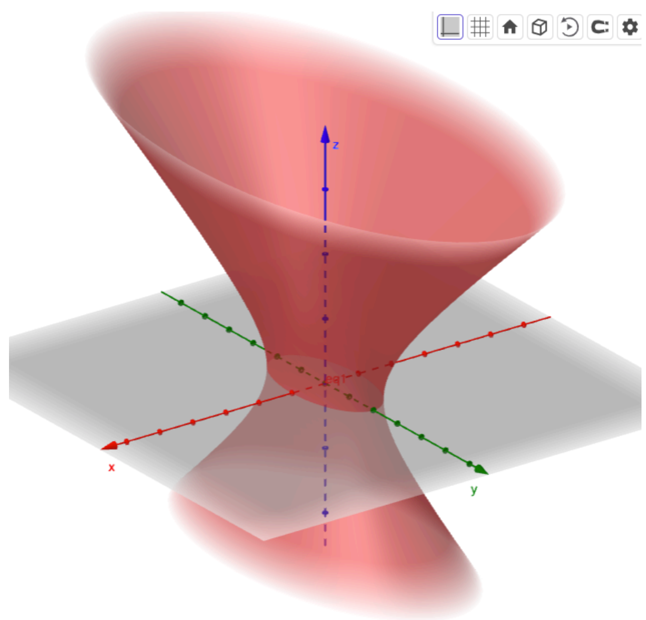
IV



V



VI



(10) Match the vector function (a-d) with the graph below (I-VI), The scale is purposely left off the graphs so do not assume scale. Two graphs have no match, you do not need to do anything with them. . Axes are standard orientation with x axis red, y axis green and z axis blue (8 points)

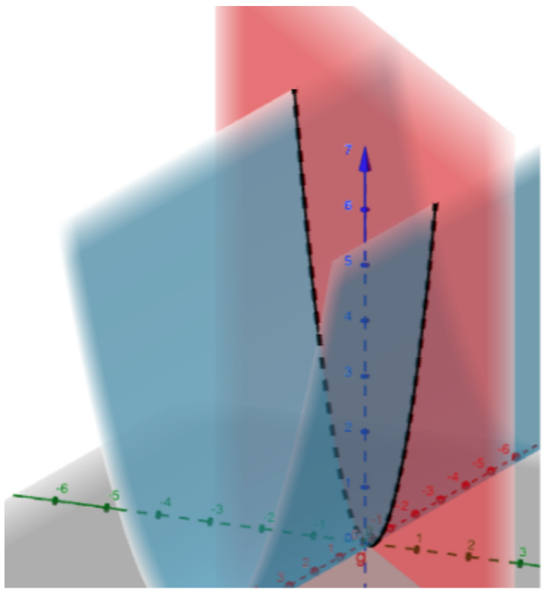
a) $\vec{r}(t)=\langle t^2, t\cos(t), t\sin(t)\rangle$ ____VI______

b) $\vec{r}(t)=\langle t^2, t^2, t^4\rangle$ ______V______

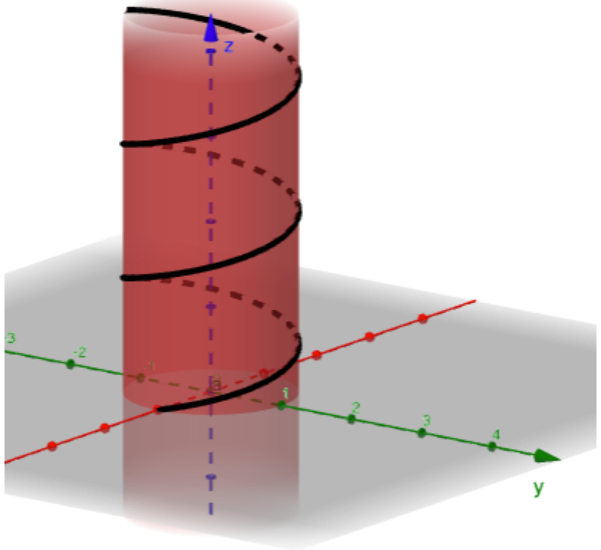
c) $\vec{r}(t)=\langle \sin(t), \cos(t), t\rangle$ ___IV__________

d) $\vec{r}(t)=\langle t, 2t, \sin(t)\rangle$ ___III________

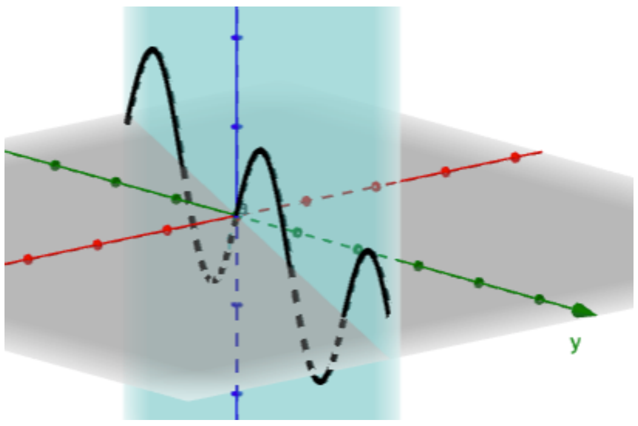
I



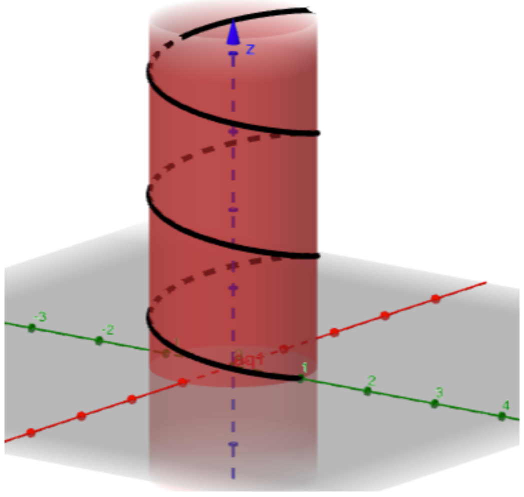
II



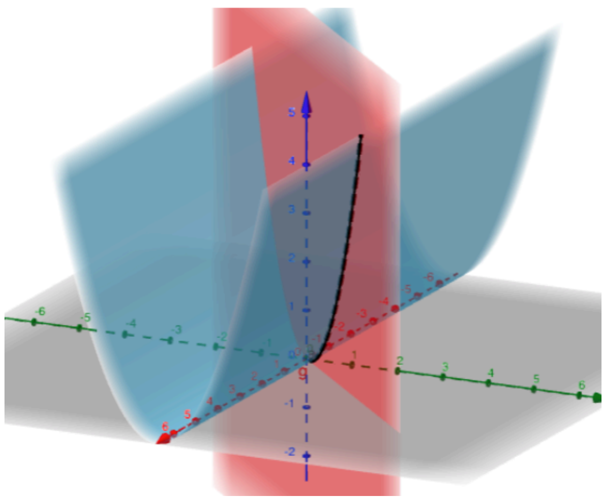
III



IV



V



VI